Outcome 1 HOMEWORK

1. Differentiate the following with respect to x:

   a) $\sin^{-1}(\cos x)$ b) $\ln x \cos^{-1} x$ c) $\tan^{-1}\left(\dfrac{2\sqrt{x}}{1-x}\right)$

2. If $x = t^2 \sin 3t$ and $y = t^2 \cos 3t$, find $\dfrac{dy}{dx}$ in terms of t, and show that the curve defined by these parametric equations is parallel to the x-axis at points where $\tan 3t = \dfrac{2}{3t}$.

3. y is a continous function of x, defined implicitly by the equation $y^2 - xy + (x^2 - 1) = 0$.

   a) If $y = 1$ when $x = 1$ find y as an explicit function of x.

   b) For what values of x is this function defined?

4. If $x^2 - 2y^2 = 2x$ find the value of

   a) $\dfrac{dy}{dx}$ b) $\dfrac{d^2y}{dx^2}$, at the point (4,2)

5. Differentiate the following with respect to x:

   a) $y = 5^x$ b) $\dfrac{x(1+x^2)^3}{(1+x^3)^{\frac{1}{3}}}$

6. Find the Cartesian equation of the curves that are defined parametrically by

   a) $x = 2\sin\theta,\ y = \cos^2\theta$ b) $x = t(t-1),\ y = 1+t$

7. A curve is given by the parametric equations:

   $x = \dfrac{(1-t)}{(1+t)},\ y = (1-t)(1+t)^2$

a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t

b) Find the equation of the tangent to the curve at the point where $t = 2$.

8. The volume, V, of a sphere of radius r, is $\frac{4\pi r^3}{3}$ and the surface area, A, is $4\pi r^2$.

The volume is increasing at a steady rate of $3\text{cm}^3/\text{s}$.

a) Find $\frac{dr}{dt}$, where t is the time in seconds.

b) Calculate the value of $\frac{dA}{dt}$ in cm^2/s at the instant when the radius is 12cm.

9. A cannon is fired horizontally from the top of a cliff. The cannonball lands 180m from the base of the cliff. If the cannonball is projected from a point 125m above the ground then $x = ut$ and $y = -5t^2$ where u is the initial velocity and x and y metres the horizontal and vertical distances of the stone from the point of projection at time t.
Find

a) the time of flight,

b) the initial velocity,

c) the speed at which the cannonball hits its target.

10. Water pours into a conical tank of semi vertical angle 30º at the rate of $4\text{cm}^3/\text{s}$, where h is the depth of the water at time t.

At what rate is the water rising in the tank when h = 10cm?